Math 111
Graphs of Functions

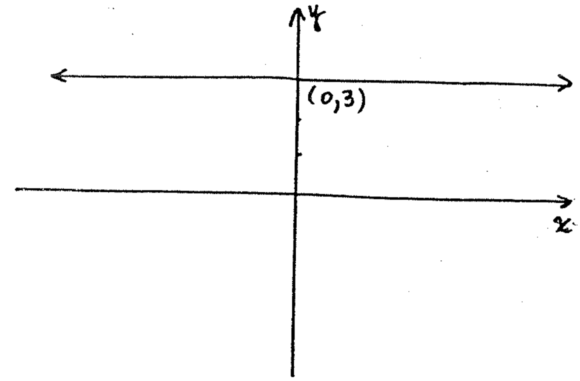
1) <u>Constant Function</u> : $f(x) = b$

Graph is a horizontal line.

<u>Example</u>. $f(x) = 3$

D: $(-\infty, \infty)$

R: $\{3\}$



2) <u>Linear Function</u> : $f(x) = mx + b \quad (m \neq 0)$

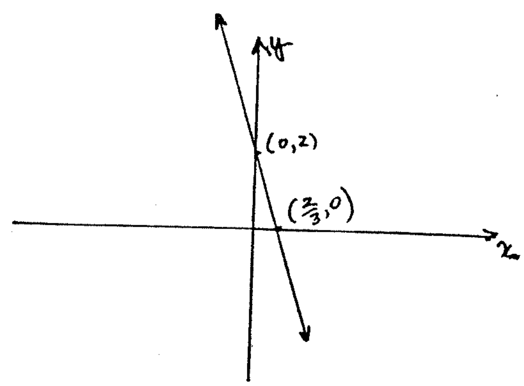
Graph is a straight line.

<u>Example</u>. $f(x) = -3x + 2$ slope $= -3$

y-int is $(0,2)$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$



3) <u>Quadratic Function</u> : $f(x) = ax^2 + bx + c \quad (a \neq 0)$

Graph is a parabola; vertex $x = -\dfrac{b}{2a}$

opens upward if $a > 0$

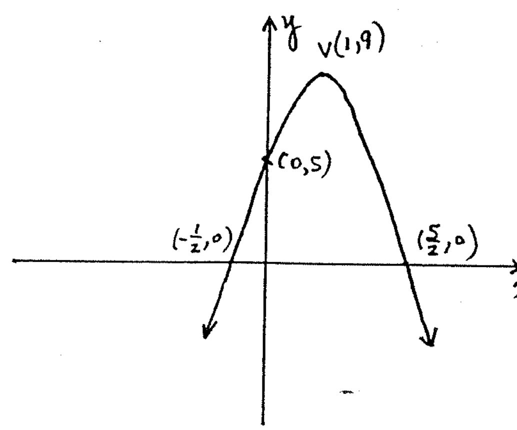
opens downward if $a < 0$

<u>Example</u>. $f(x) = -4x^2 + 8x + 5$

vertex : $(1, 9)$; opens downward

D: $(-\infty, \infty)$

R: $(-\infty, 9]$



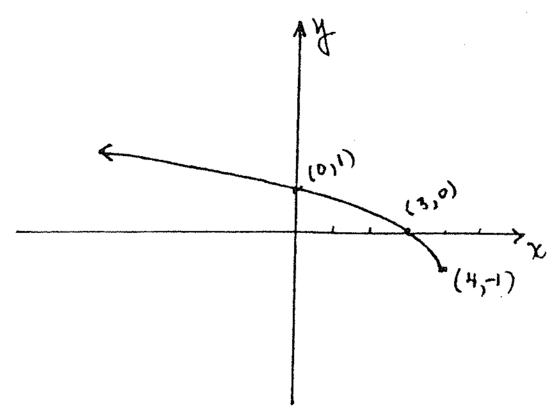
4) <u>Functions involving square roots</u>

(a) half-parabolas : $f(x) = c \pm \sqrt{ax+b}$

<u>Example</u>. $f(x) = -1 + \sqrt{4-x}$

D: $(-\infty, 4]$

R: $[1, \infty)$



(b) <u>Semicircles</u> : $f(x) = b \pm \sqrt{a^2-x^2}$

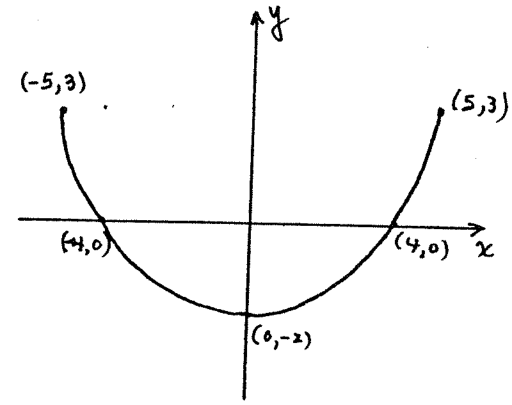
part of circle with center $(0,b)$ and radius $r = a$

<u>Example</u>. $f(x) = 3 - \sqrt{25-x^2}$

bottom half of circle $C(0,3)$ and radius $r = 5$

D: $[-5,5]$

R: $[-2,3]$

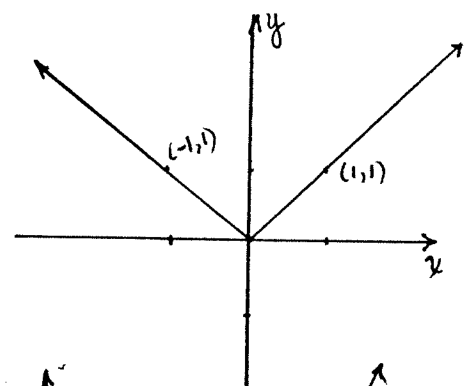


5) <u>Piecewise-defined functions</u> : The rule defining the function consists of more than one formula.

(a) <u>Absolute Value</u>: $f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

D: $(-\infty, \infty)$

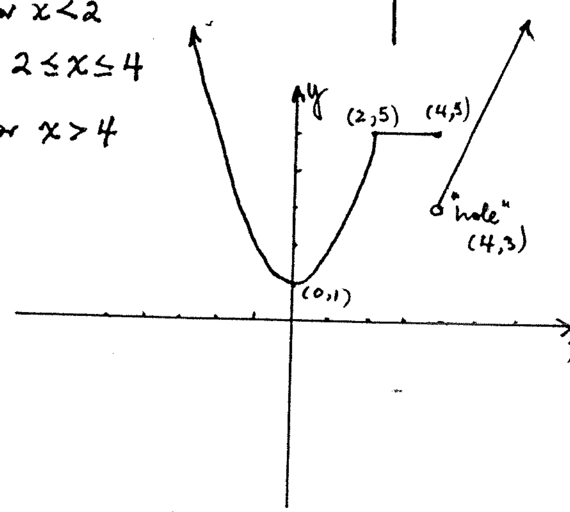
R: $[0, \infty)$



(b) <u>Another example</u>: $g(x) = \begin{cases} x^2+1, & \text{for } x < 2 \\ 5, & \text{for } 2 \leq x \leq 4 \\ 2x-5, & \text{for } x > 4 \end{cases}$

D: $(-\infty, \infty)$

R: $[1, \infty)$

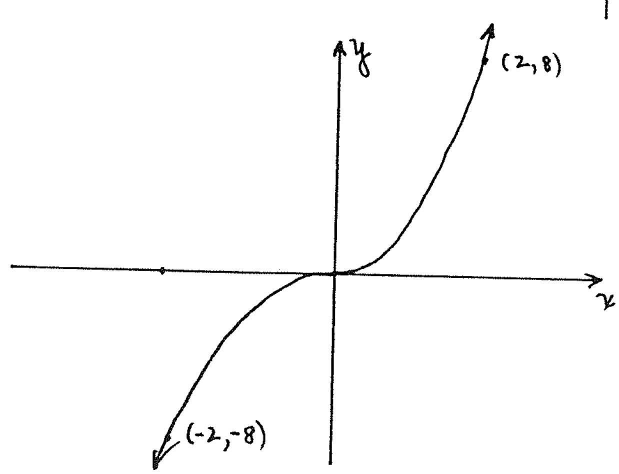


6) <u>Some additional algebraic functions</u>

(a) $f(x) = x^3$

D: $(-\infty, \infty)$

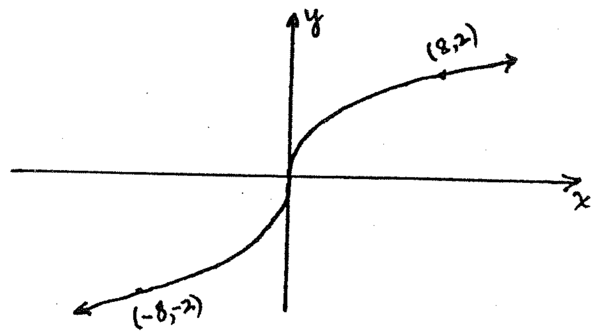
R: $(-\infty, \infty)$



(b) $f(x) = \sqrt[3]{x} = x^{1/3}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

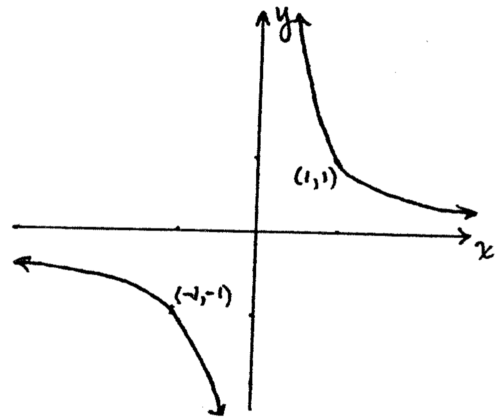


(c) $f(x) = \dfrac{1}{x}$

D: $x \neq 0$, or $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

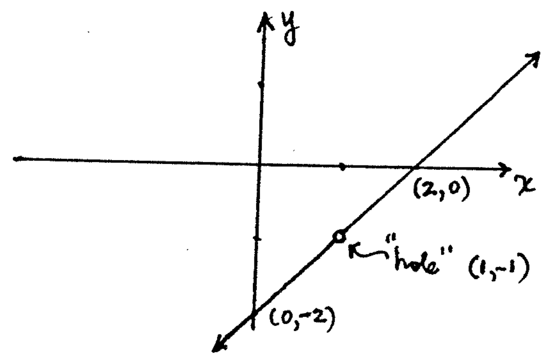
asymptotes: $x = 0$ and $y = 0$



(d) $f(x) = \dfrac{x^2 - 3x + 2}{x - 1}$

D: $(-\infty, 1) \cup (1, \infty)$

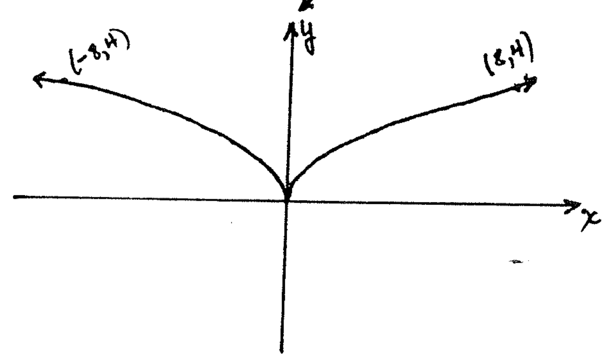
R: $(-\infty, -1) \cup (-1, \infty)$



(e) $f(x) = x^{2/3}$

D: $(-\infty, \infty)$

R: $[0, \infty)$



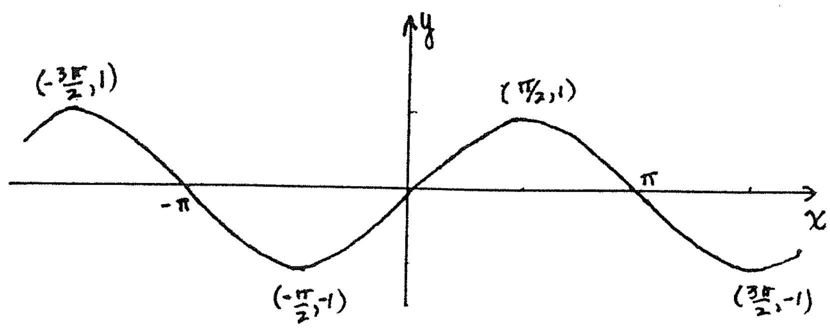
7) <u>Trigonometric Functions</u>

(a) $f(x) = \sin x$

D: $(-\infty, \infty)$

R: $[-1, 1]$

period: 2π

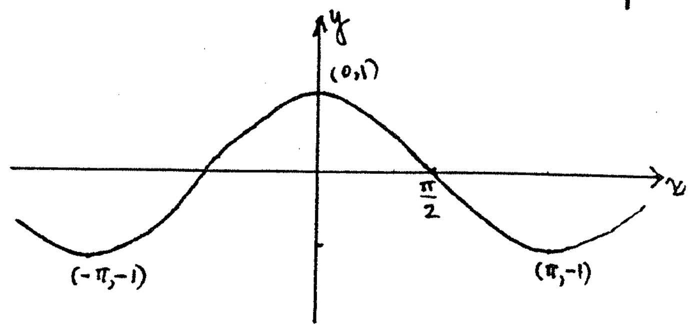


(b) $f(x) = \cos x$

$D: (-\infty, \infty)$

$R: [-1, 1]$

period : 2π



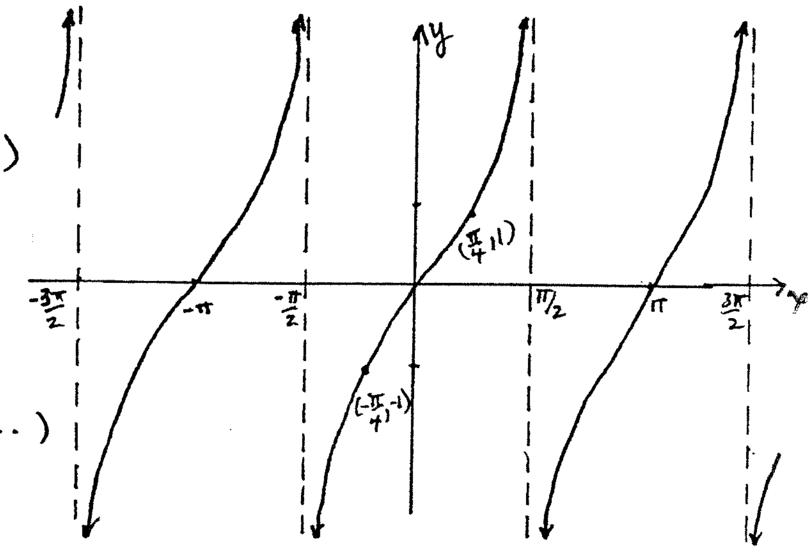
(c) $f(x) = \tan x$

$D: x \neq \pm(2n+1)\frac{\pi}{2}$
$(n = 0, 1, 2, \cdots)$

$R: (-\infty, \infty)$

period : π

asymptotes : $x = \pm(2n+1)\frac{\pi}{2}$
$(n = 0, 1, 2, \cdots)$



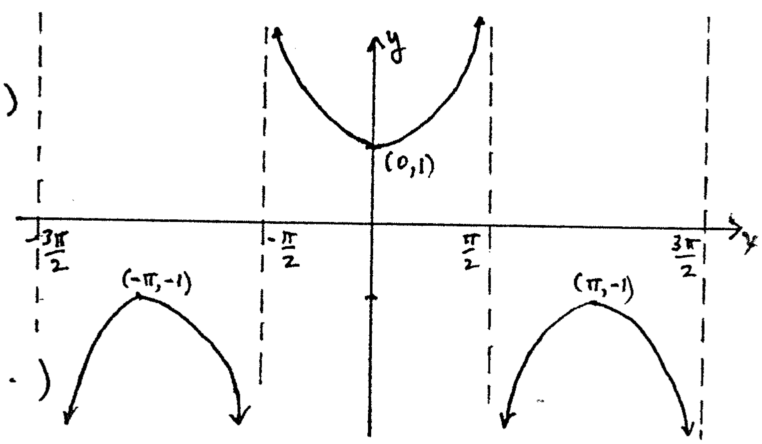
(d) $f(x) = \sec x$

$D: x \neq \pm(2n+1)\frac{\pi}{2}$
$(n = 0, 1, 2, \cdots)$

$R: (-\infty, -1] \cup [1, \infty)$

period : 2π

asymptotes : $x = \pm(2n+1)\frac{\pi}{2}$
$(n = 0, 1, 2, \cdots)$



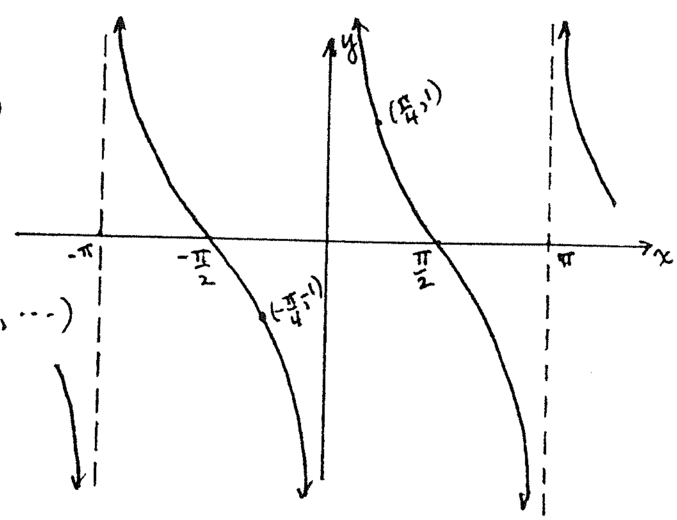
(e) $f(x) = \cot x$

$D: x \neq n\pi \quad (n = 0, 1, 2, \cdots)$

$R: (-\infty, \infty)$

period : π

asymptotes : $x = \pm n\pi \; (n = 0, 1, 2, \cdots)$



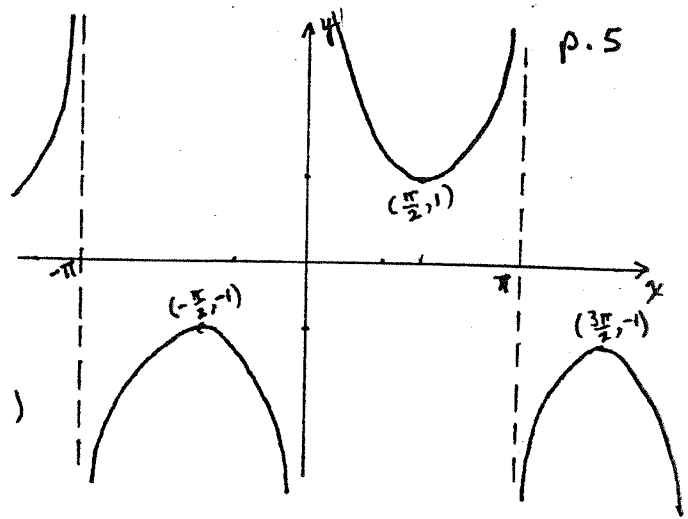
(f) $f(x) = \csc x$

D: $x \neq n\pi \quad (n = 0, 1, 2, \cdots)$

R: $(-\infty, -1] \cup [1, \infty)$

period: 2π

asymptotes: $x = \pm n\pi \quad (n = 0, 1, 2, \cdots)$



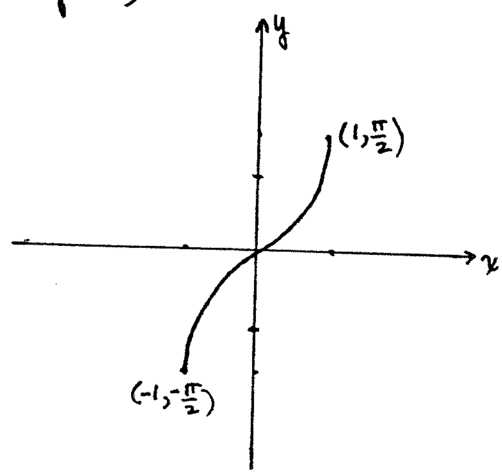
8) Inverse Trigonometric Functions (Three Examples)

(a) $f(x) = \text{Arcsin}\, x$

$y = \text{Arcsin}\, x$ iff $x = \sin y$
and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

D: $[-1, 1]$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

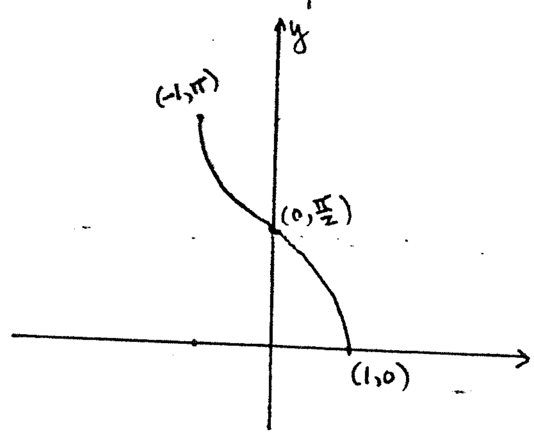


(b) $f(x) = \text{Arccos}\, x$

$y = \text{Arccos}\, x$ iff $x = \cos y$
and $0 \le y \le \pi$

D: $[-1, 1]$

R: $[0, \pi]$



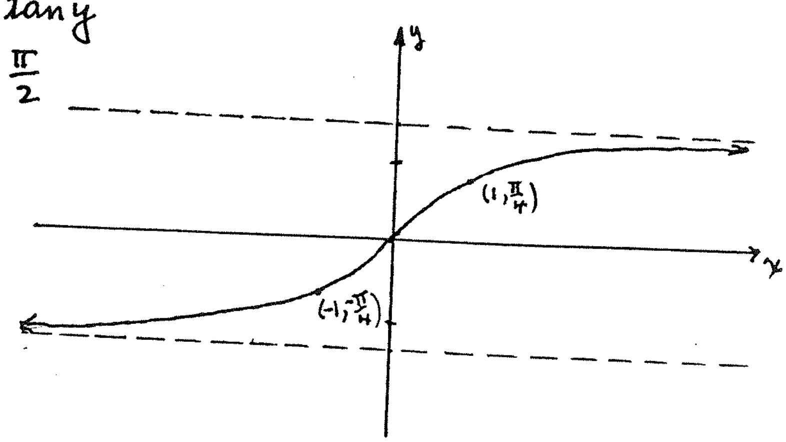
(c) $f(x) = \text{Arctan}\, x$

$y = \text{Arctan}\, x$ iff $x = \tan y$
and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

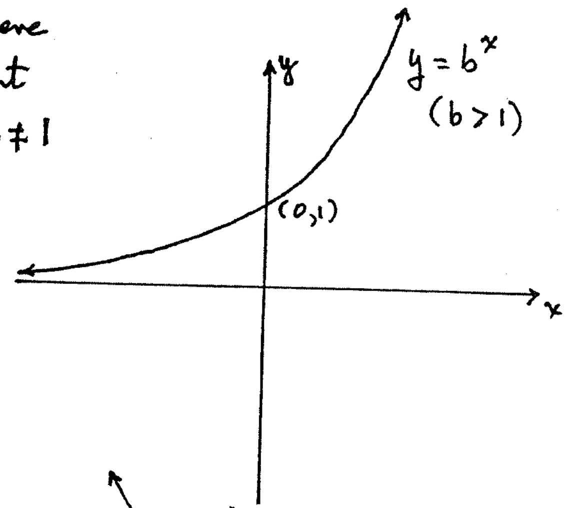
D: $(-\infty, \infty)$

R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

asymptotes: $y = -\frac{\pi}{2}$ &
$y = \frac{\pi}{2}$



(9) <u>Exponential Functions</u> : $f(x) = b^x$, where b is a constant and $b > 0, b \neq 1$



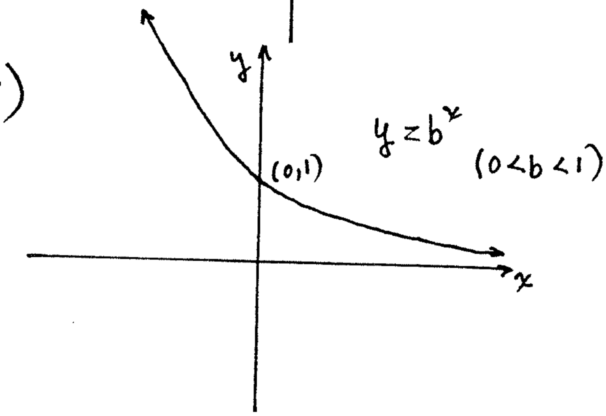
D: $(-\infty, \infty)$

R: $(0, \infty)$

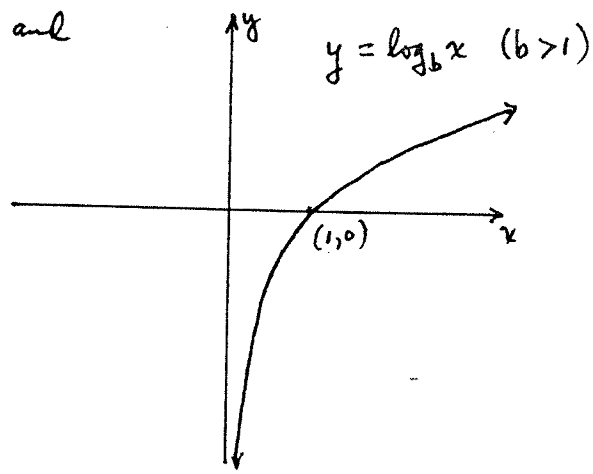
asymptote : $y = 0$

<u>Examples</u> : $f(x) = e^x \quad (e > 1)$

$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x \quad \left(0 < \frac{1}{2} < 1\right)$



(10) <u>Logarithmic Functions</u> : $f(x) = \log_b x$, where b is a constant and $b > 0, \ b \neq 1$



$y = \log_b x$ iff $x = b^y$

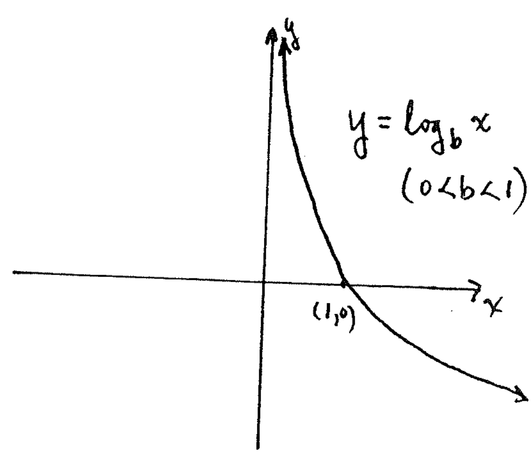
D: $(0, \infty)$

R: $(-\infty, \infty)$

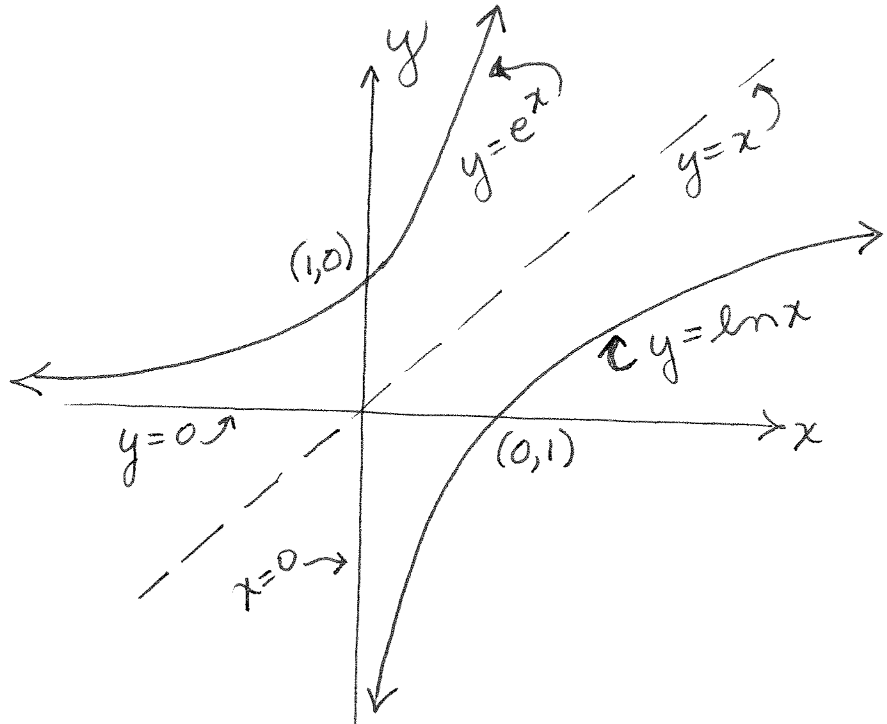
asymptote : $x = 0$

<u>Examples</u> : $f(x) = \ln x = \log_e x \quad (e > 1)$

$f(x) = \log_{\frac{1}{2}} x \qquad \left(0 < \frac{1}{2} < 1\right)$



Natural Logarithmic and Exponential Functions



Inverse Relations

$y = e^x$ iff $x = \ln y$

$\ln e^A = A$

$e^{\ln A} = A$

Definition

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

or e is the number such that $\ln e = 1$

($e \approx 2.718$)

Logarithm

$y = \ln x$, $x > 0$ (all y)

$\ln(ab) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^r = r \ln a$

$\ln 1 = 0$

Exponential

$y = e^x$, all x ($y > 0$)

$e^a e^b = e^{a+b}$

$\frac{e^a}{e^b} = e^{a-b}$

$(e^a)^r = e^{ra}$

Notes:

1.) Composition of functions:

$(f\circ g)(x) = f(g(x)) \qquad (g\circ f)(x) = g(f(x))$

ex

$f(x) = 3x^2+2 \quad g(x) = \sqrt[3]{x}$

$(f\circ g)(x) = f(\sqrt[3]{x}) = 3(\sqrt[3]{x})^2+2 = 3x^{2/3}+2$

$(g\circ f)(x) = g(f(x)) = g(3x^2+2) = (3x^2+2)^{1/3}$

2.) $\boxed{180^\circ = \pi \text{ radians}}$

Identities:

$\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x}$

$\sec x = \dfrac{1}{\cos x} \qquad \csc x = \dfrac{1}{\sin x}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

3.) Shifts in graphs.

$y = f(x) + c$ — for $c>0$, shifts $f(x)$ up y-axis "c" units
for $c<0$, shifts $f(x)$ down y-axis "c" units

$y = f(x+c)$ — for $c>0$, shifts $f(x)$ left "c" units along x-axis
for $c<0$, shifts $f(x)$ right "c" units along x-axis