2a $3x^2+6=10x$

$\underbrace{3x^2-10x+6}_{\text{call this } f(x).}=0$

Let $g(x)=f(x+h)$

$= 3(x+h)^2-10(x+h)+6$

$= 3(x^2+2xh+h^2) - 10x - 10h + 6$

$= 3x^2+6xh+3h^2-10x-10h+6$

$= 3x^2+\underbrace{(6h-10)}_{}x + (3h^2-10h+6)$

we want this to vanish, so choose $h=\frac{5}{3}$ (ie the solution to $6h-10=0$)

So for $h=\frac{5}{3}$,

$g(x) = 3x^2+\left(3\left(\frac{5}{3}\right)^2-10\left(\frac{5}{3}\right)+6\right)$

$= 3x^2+\frac{25}{3}-\frac{50}{3}+\frac{18}{3}$

$= 3x^2-\frac{7}{3}$

We can find solutions to $g(x)=0$ with "socks and shoes"

$3x^2-\frac{7}{3}=0$

$3x^2=\frac{7}{3}$

$x^2=\frac{7}{9}$

$x=\frac{\pm\sqrt{7}}{3}$

Finally, as g is a horizontal shift of f left by h, we can "shift back" the solutions to $g(x)=0$ by adding $h=\frac{5}{3}$ to each to find solutions to the original equation $f(x)=0$

$x=\frac{5}{3}\pm\frac{\sqrt{7}}{3}$

$$\boxed{x=\frac{5\pm\sqrt{7}}{3}}$$

2b $\underbrace{3t^2+8t+3}_{\text{call this } f(t)}=0$

$g(x)=f(x+h)$

$=3(x+h)^2+8(x+h)+3$

$=3(x^2+2xh+h^2)+8x+8h+3$

$=3x^2+6xh+3h^2+8x+8h+3$

$=3x^2+\underbrace{(6h+8)}_{}x+(3h^2+8h+3)$

Let $h=-\frac{8}{6}=\frac{-4}{3}$ so this term vanishes (ie. gets depressed)

So for $h=-\frac{4}{3}$

$g(x)=3x^2+\left(3\left(\frac{-4}{3}\right)^2+8\left(\frac{-4}{3}\right)+3\right)$

$=3x^2+\left(\frac{16}{3}-\frac{32}{3}+\frac{9}{3}\right)$

$=3x^2-\frac{7}{3}$

Solving $g(x)=0$ we have

$3x^2-\frac{7}{3}=0$

$3x^2=\frac{7}{3}$

$x^2=\frac{7}{9}$

$x=\frac{\pm\sqrt{7}}{3}$

Which means $f(x)=0$ is solved by

$x=\frac{-4}{3}\pm\frac{\sqrt{7}}{3}$

$$\boxed{x=\frac{-4\pm\sqrt{7}}{3}}$$

2c $5t^2-8t=3$

$\underbrace{5t^2-8t-3}_{f(t)}=0$

$g(x)=f(x+h)$

$=5(x+h)^2-8(x+h)-3$

$=5(x^2+2xh+h^2)-8x-8h-3$

$=5x^2+10xh+5h^2-8x-8h-3$

$=5x^2+\underbrace{(10h-8)}_{\text{choose } h=\frac{8}{10}=\frac{4}{5} \text{ to depress this term}}x+(5h^2-8h-3)$

For $h=\frac{4}{5}$

$g(x)=5x^2+\left(5\left(\frac{4}{5}\right)^2-8\left(\frac{4}{5}\right)-3\right)$

$=5x^2+\left(\frac{16}{5}-\frac{32}{5}-\frac{15}{5}\right)$

$=5x^2-\frac{31}{5}$

Solving $g(x)=0$ ...

$5x^2-\frac{31}{5}=0$

$5x^2=\frac{31}{5}$

$x^2=\frac{31}{25}$

$x=\pm\frac{\sqrt{31}}{5}$

So the solutions to $f(x)=0$ are given by

$x=\frac{4}{5}\pm\frac{\sqrt{31}}{5}$

$\boxed{x=\frac{4\pm\sqrt{31}}{5}}$

2d $5m^2+3m=2$

$\underbrace{5m^2+3m-2}_{f(m)}=0$

$g(x)=f(x+h)$

$=5(x+h)^2+3(x+h)-2$

$=5(x^2+2xh+h^2)+3x+3h-2$

$=5x^2+10xh+5h^2+3x+3h-2$

$=5x^2+\underbrace{(10h+3)}_{h=\frac{-3}{10}}x+(5h^2+3h-2)$

For $h=\frac{-3}{10}$,

$g(x)=5x^2+\left(5\left(\frac{-3}{10}\right)^2+3\left(\frac{-3}{10}\right)-2\right)$

$=5x^2+\left(\frac{9}{20}-\frac{9}{10}-2\right)$

$=5x^2+\left(\frac{9}{20}-\frac{18}{20}-\frac{40}{20}\right)$

$=5x^2+\left(\frac{-49}{20}\right)$

$=5x^2-\frac{49}{20}$

Solving $g(x)=0$ ...

$5x^2-\frac{49}{20}=0$

$5x^2=\frac{49}{20}$

$x^2=\frac{49}{100}$

$x=\pm\frac{7}{10}$

So $f(x)=0$ solved by

$x=\frac{-3}{10}\pm\frac{7}{10}$

$x=\frac{-10}{10}$ or $\frac{4}{10}$

$\boxed{x=-1 \text{ or } \frac{2}{5}}$

2e $\underbrace{x^2-6x+3}_{f(x)}=0$

$g(x)=f(x+h)$

$=(x+h)^2-6(x+h)+3$

$=x^2+2xh+h^2-6x-6h+3$

$=x^2+\underbrace{(2h-6)}_{h=3}x+(h^2-6h+3)$

So for $h=3$

$g(x)=x^2+(9-18+3)$

$=x^2-6$

Solving $g(x)=0$

$x^2-6=0$

$x^2=6$

$x=\pm\sqrt{6}$

solutions to $f(x)=0$ are:

$\boxed{x=3\pm\sqrt{6}}$