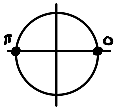
1a $\tan x = 0$

$\frac{\sin x}{\cos x} = 0$

$\sin x = 0$ (and $\cos x \neq 0$)



$\boxed{x = \begin{cases} 0 + 2\pi n \\ \pi + 2\pi n \end{cases}, \; n \in \mathbb{Z}}$

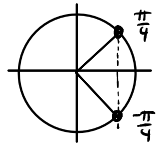
or more compactly,

$\boxed{x = \pi n, \; n \in \mathbb{Z}}$

1b $2\cos x + \sqrt{2} = 0$

$2\cos x = -\sqrt{2}$

$\cos x = \frac{-\sqrt{2}}{2}$

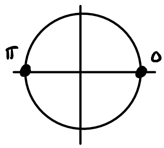


$\boxed{x = \pm\frac{\pi}{4} + 2\pi n, \; n \in \mathbb{Z}}$

1c $\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$



$x = \begin{cases} 0 + 2\pi n \\ \pi + 2\pi n \end{cases}, \; n \in \mathbb{Z}$

or more compactly,

$\boxed{x = \pi n, \; n \in \mathbb{Z}}$

1d

$2\cos^2 x - 3\cos x - 2 = 0$

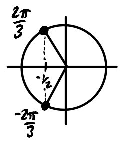
Let $u = \cos x$

$2u^2 - 3u - 2 = 0$

$(2u+1)(u-2) = 0$

$u = -\frac{1}{2}$ or $u = 2$

$\cos x = -\frac{1}{2}$ or $\cancel{\cos x = 2}$ impossible



$\boxed{x = \pm\frac{2\pi}{3} + 2\pi n, \; n \in \mathbb{Z}}$

1e $\tan^2 x + (\sqrt{3} - 1)\tan x - \sqrt{3} = 0$

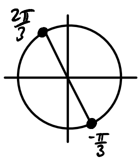
Let $u = \tan x$

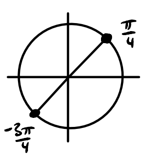
$u^2 + (\sqrt{3} - 1)u - \sqrt{3} = 0$

$(u + \sqrt{3})(u - 1) = 0$

$u = -\sqrt{3}$ or $u = 1$

$\tan x = -\sqrt{3}$ or $\tan x = 1$





$\boxed{x = \begin{cases} \frac{2\pi}{3} + \pi n \\ \frac{\pi}{4} + \pi n \end{cases}, \; n \in \mathbb{Z}}$

1f

$3\sec^2 x = \sec x$

Let $u = \sec x$

$3u^2 = u$

$3u^2 - u = 0$

$u(3u - 1) = 0$

$u = 0$ or $u = \frac{1}{3}$

$\cancel{\sec x = 0}$ or $\cancel{\sec x = \frac{1}{3}}$

Both impossible as $|\sec\theta| \geq 1$ always

$\boxed{\text{no solution}}$

1g

$2\sin^2 x - \sin x - 1 = 0$

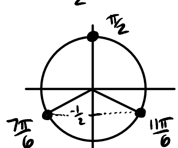
Let $u = \sin x$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$u = -\frac{1}{2}$ or $u = 1$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$



$\boxed{x = \frac{\pi}{2} + \frac{2\pi}{3}n \text{ when } n \in \mathbb{Z}}$

1h $\cos 2x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

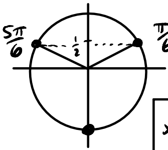
Let $u = \sin x$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2}$ or $u = -1$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

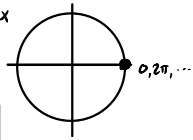


$$\boxed{x = \frac{\pi}{6} + \frac{2\pi}{3}n \text{ where } n \in \mathbb{Z}}$$

1i $\dfrac{1+\cos x}{\cos x} = 2$

$1 + \cos x = 2\cos x$

$1 = \cos x$



$$\boxed{x = 2\pi n,\ n \in \mathbb{Z}}$$

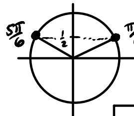
1j $\sqrt{\dfrac{1+2\sin x}{2}} = 1$

$\dfrac{1+2\sin x}{2} = 1$

$1 + 2\sin x = 2$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$



$$\boxed{x = \begin{cases} \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \end{cases},\ n \in \mathbb{Z}}$$

1k $\cos^3 x - \cos x = 0$

Let $u = \cos x$

$u^3 - u = 0$

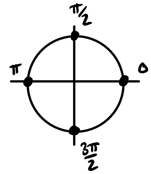
$u(u^2 - 1) = 0$

$u(u+1)(u-1) = 0$

$u = 0$ → $\cos x = 0$

$u = -1$ → $\cos x = -1$

$u = 1$ → $\cos x = 1$



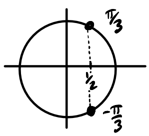
$$\boxed{x = \begin{cases} 0 + 2\pi k \\ \frac{\pi}{2} + 2\pi k \\ \pi + 2\pi k \\ \frac{3\pi}{2} + 2\pi k \end{cases} \text{ where } k \in \mathbb{Z}}$$

Taking advantage of the symmetry here, we can also write the solution more tightly (less ink) as:

$$\boxed{x = \frac{\pi}{2}k,\ k \in \mathbb{Z}}$$

1l $2\cos 3x = 1$

$\cos 3x = \frac{1}{2}$



$3x = \pm\frac{\pi}{3} + 2\pi n,\ n \in \mathbb{Z}$

$$\boxed{x = \pm\frac{\pi}{9} + \frac{2}{3}\pi n,\ n \in \mathbb{Z}}$$

1m $\cos^2\theta + \sin\theta = \frac{5}{4}$

$(1 - \sin^2\theta) + \sin\theta = \frac{5}{4}$

Let $u = \sin\theta$

$1 - u^2 + u = \frac{5}{4}$

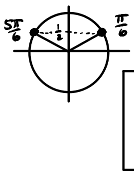
$u^2 - u + \frac{1}{4} = 0$

$4u^2 - 4u + 1 = 0$

$(2u-1)^2 = 0$

$u = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$



$$\boxed{\theta = \begin{cases} \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \end{cases},\ n \in \mathbb{Z}}$$

1n $2\sec^2 x - 5\tan x - 3 = 0$

$2(1 + \tan^2 x) - 5\tan x - 3 = 0$

Let $u = \tan x$

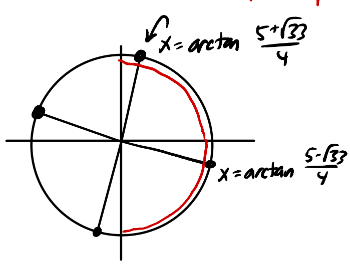
$2(1 + u^2) - 5u - 3 = 0$

$2u^2 - 5u - 1 = 0$

$u = \dfrac{5 \pm \sqrt{25+8}}{4} = \dfrac{5 \pm \sqrt{33}}{4}$

$\tan x = \dfrac{5 \pm \sqrt{33}}{4}$ ← note, these values of tangent do not correspond to "nice" angles that are a multiple of 30° or 45°

so we use arctan to get one solution and symmetry/periodicity to get another (for each)



$$x = \begin{cases} \arctan\left(\frac{5+\sqrt{33}}{4}\right) + \pi n \\ \arctan\left(\frac{5-\sqrt{33}}{4}\right) + \pi n \end{cases},\ n \in \mathbb{Z}$$