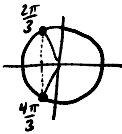


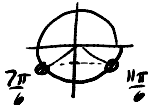
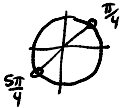
5a) $\tan^2 x + \sec^2 x + 3 \sec x = 1$
 $(\sec^2 - 1) + \sec^2 x + 3 \sec x = 1$
 $2 \sec^2 x + 3 \sec x - 2 = 0$
Let $u = \sec x$
 $2u^2 + 3u - 2 = 0$
 $(2u - 1)(u + 2) = 0$
 $u = \frac{1}{2}$ or $u = -2$
 $\sec x = \frac{1}{2}$ or $\sec x = -2$
 ~~$\cos x = 2$ or $\cos x = -\frac{1}{2}$~~



$$x = \begin{cases} \frac{2\pi}{3} + 2n\pi \\ \frac{4\pi}{3} + 2n\pi \end{cases}$$

where $n \in \mathbb{Z}$

5c) $2 \sin x \tan x + \tan x - 2 \sin x - 1 = 0$
 $\tan x (2 \sin x + 1) - (2 \sin x + 1) = 0$
 $(\tan x - 1)(2 \sin x + 1) = 0$
 $\tan x = 1$ or $\sin x = -\frac{1}{2}$



$$x = \begin{cases} \frac{\pi}{4} + 2n\pi \\ \frac{5\pi}{4} + 2n\pi \\ \frac{7\pi}{6} + 2n\pi \\ \frac{11\pi}{6} + 2n\pi \end{cases}$$

where $n \in \mathbb{Z}$

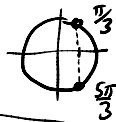
5b) $\cos 2x = 1 + \sin x$
 $1 - 2 \sin^2 x = 1 + \sin x$
 $1 - 2u^2 = 1 + u$
 $2u^2 + u = 0$
 $u(2u + 1) = 0$
 $u = 0$ or $u = -\frac{1}{2}$
 $\sin x = 0$ or $\sin x = -\frac{1}{2}$



$$x = \begin{cases} 0 + 2n\pi \\ \pi + 2n\pi \\ \frac{7\pi}{6} + 2n\pi \\ \frac{11\pi}{6} + 2n\pi \end{cases}$$

where $n \in \mathbb{Z}$

5d) $2 \cos^2 x - 3 \cos x + 1 = 0$
Let $u = \cos x$
 $2u^2 - 3u + 1 = 0$
 $(2u - 1)(u - 1) = 0$
 $u = \frac{1}{2}$ or $u = 1$
 $\cos x = \frac{1}{2}$ or $\cos x = 1$



$$x = \begin{cases} \frac{\pi}{3} + 2n\pi \\ \frac{5\pi}{3} + 2n\pi \\ 0 + 2n\pi \end{cases}$$

where $n \in \mathbb{Z}$

$$(52) 3 \tan x + \frac{1}{\tan x} = 2\sqrt{3}$$

$$\text{Let } u = \tan x$$

$$3u + \frac{1}{u} = 2\sqrt{3}$$

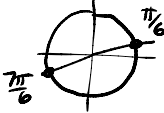
$$3u^2 + 1 = 2\sqrt{3} \cdot u$$

$$3u^2 - 2\sqrt{3}u + 1 = 0$$

$$(3u-1)^2 = 0$$

$$u = \frac{1}{\sqrt{3}}$$

$$\tan x = \frac{1}{\sqrt{3}}$$



$$x = \begin{cases} \frac{\pi}{6} + 2n\pi \\ \frac{7\pi}{6} + 2n\pi \end{cases} \text{ where } n \in \mathbb{Z}$$

$$(53) 2 \sin x \cdot \tan x = 3$$

$$2 \sin x \cdot \frac{\sin x}{\cos x} = 3$$

$$\frac{2 \sin^2 x}{\cos x} = 3$$

$$2(1 - \cos^2 x) = 3 \cos x$$

$$\text{Let } u = \cos x$$

$$2(1 - u^2) = 3u$$

$$2(1 - u^2) = 3u$$

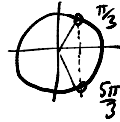
$$-2u^2 + 2 = 3u$$

$$2u^2 + 3u - 2 = 0$$

$$(2u-1)(u+2) = 0$$

$$u = \frac{1}{2} \text{ or } u = -2$$

$$\cos x = \frac{1}{2} \text{ or } \cos x = -2$$



$$x = \begin{cases} \frac{\pi}{3} + 2n\pi \\ \frac{5\pi}{3} + 2n\pi \end{cases} \text{ where } n \in \mathbb{Z}$$

$$(54) \sec^2 x - \tan x = 1$$

$$(1 + \tan^2 x) - \tan x = 1$$

$$\text{Let } u = \tan x$$

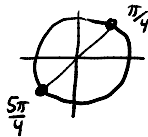
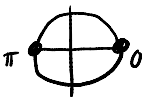
$$1 + u^2 - u = 1$$

$$u^2 - u = 0$$

$$u(u-1) = 0$$

$$u = 0 \text{ or } u = 1$$

$$\tan x = 0 \text{ or } \tan x = 1$$



$$x = \begin{cases} 0 + 2n\pi \\ \pi + 2n\pi \\ \frac{\pi}{4} + 2n\pi \\ \frac{5\pi}{4} + 2n\pi \end{cases} \text{ where } n \in \mathbb{Z}$$

$$(55) 4 \sin^2 x - 1 = 0$$

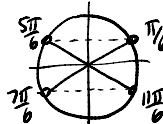
$$\text{Let } u = \sin x$$

$$4u^2 - 1 = 0$$

$$(2u+1)(2u-1) = 0$$

$$u = -\frac{1}{2} \text{ or } u = \frac{1}{2}$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = \frac{1}{2}$$



$$x = \begin{cases} \frac{\pi}{6} + 2n\pi \\ \frac{5\pi}{6} + 2n\pi \\ \frac{7\pi}{6} + 2n\pi \\ \frac{11\pi}{6} + 2n\pi \end{cases} \text{ where } n \in \mathbb{Z}$$

(51) $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

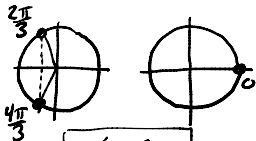
Let $u = \cos x$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$u = -\frac{1}{2}$ or $u = 1$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$



$x = \begin{cases} 0 + 2\pi n \\ \frac{2\pi}{3} + 2\pi n \\ \frac{4\pi}{3} + 2\pi n \end{cases}$
where $n \in \mathbb{Z}$

(52)

$2\cos^2 x + \sin^2 x = 1$

$2\cos^2 x + (1 - \cos^2 x) = 1$

Let $u = \cos x$

$2u^2 + 1 - u^2 = 1$

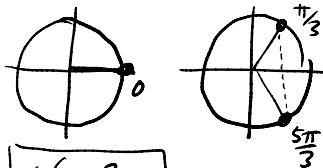
$u^2 = 0$

$u(2u-1) = 0$

$u^2 = 0$ or $2u-1 = 0$

$u = 0$ or $u = \frac{1}{2}$

$\cos x = 0$ or $\cos x = \frac{1}{2}$



$x = \begin{cases} 0 + 2\pi n \\ \frac{\pi}{3} + 2\pi n \\ \frac{5\pi}{3} + 2\pi n \end{cases}$
where $n \in \mathbb{Z}$

(53) $8\sin^4 x - 10\sin^2 x + 3 = 0$

Let $u = \sin x$

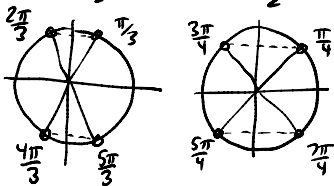
$8u^4 - 10u^2 + 3 = 0$

$(4u^2 - 3)(2u^2 - 1) = 0$

$u^2 = \frac{3}{4}$ or $u^2 = \frac{1}{2}$

$u = \pm\frac{\sqrt{3}}{2}$ or $u = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\sin x = \pm\frac{\sqrt{3}}{2}$ or $\sin x = \pm\frac{\sqrt{2}}{2}$



$x = \begin{cases} \frac{\pi}{3} + \pi n \\ \frac{2\pi}{3} + \pi n \\ \frac{\pi}{4} + \pi n \\ \frac{3\pi}{4} + \pi n \end{cases}$
where $n \in \mathbb{Z}$

(54) $2\sin^2 x + 7\sin x + 3 = 0$

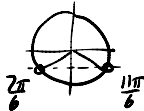
Let $u = \sin x$

$2u^2 + 7u + 3 = 0$

$(2u+1)(u+3) = 0$

$u = -\frac{1}{2}$ or $u = -3$

$\sin x = -\frac{1}{2}$ or $\sin x = -3$



$x = \begin{cases} \frac{7\pi}{6} + 2\pi n \\ \frac{11\pi}{6} + 2\pi n \end{cases}$
where $n \in \mathbb{Z}$

(55)

$3\cot x = \tan x$

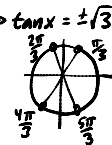
$\frac{3}{\tan x} = \tan x$

Let $u = \tan x$

$\frac{3}{u} = u$

$3 = u^2$

$u = \pm\sqrt{3}$



$x = \begin{cases} \frac{\pi}{3} + \pi n \\ \frac{2\pi}{3} + \pi n \end{cases}$
where $n \in \mathbb{Z}$