6a

$$\frac{1}{\tan x+\cot x} \stackrel{?}{=} (\sin x)(\cos x)$$

$$= \frac{1}{\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)} \cdot \frac{(\cos x)(\sin x)}{(\cos x)(\sin x)}$$

$$= \frac{(\cos x)(\sin x)}{\sin^2 x+\cos^2 x}$$

$$= \frac{(\sin x)(\cos x)}{1} =$$

✓

6b

$$\sec^2\alpha+\csc^2\alpha \stackrel{?}{=} \tan^2\alpha+\cot^2\alpha+2$$

$$= \tan^2\alpha+1+\cot^2\alpha+1$$

$$= \tan^2\alpha+\cot^2\alpha+2 =$$

✓

6c

$$\frac{\tan x-\sin x}{\tan x+\sin x} \stackrel{?}{=} \frac{\sec x-1}{\sec x+1}$$

$$= \frac{\left(\frac{\sin x}{\cos x}-\sin x\right)}{\left(\frac{\sin x}{\cos x}+\sin x\right)} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\sin x-(\sin x)(\cos x)}{\sin x+(\sin x)(\cos x)}$$

$$= \frac{\cancel{(\sin x)}(1-\cos x)}{\cancel{(\sin x)}(1+\cos x)} =$$

$$= \frac{\frac{1}{\cos x}-1}{\frac{1}{\cos x}+1}$$

$$= \frac{1-\cos x}{1+\cos x}$$

✓

6d

$$(1-\cos^2\theta)(1+\cot^2\theta) \stackrel{?}{=} 1$$

$$= \sin^2\theta\,(1+\cot^2\theta)$$

$$= \sin^2\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)$$

$$= \sin^2\theta+\cos^2\theta$$

$$= 1 =$$

✓

6e

$$\frac{\csc\beta-\sin\beta}{1-\sin^2\beta} \stackrel{?}{=} \csc\beta$$

$$= \frac{\left(\frac{1}{\sin\beta}-\sin\beta\right)}{1-\sin^2\beta} \cdot \frac{\sin\beta}{\sin\beta}$$

$$= \frac{\cancel{1-\sin^2\beta}}{\cancel{(1-\sin^2\beta)}\cdot\sin\beta}$$

$$= \frac{1}{\sin\beta} =$$

$$= \frac{1}{\sin\beta}$$

✓

6f

$$\frac{\sin\gamma\sec^2\gamma-\sin\gamma}{\cos\gamma} \stackrel{?}{=} \tan^3\gamma$$

$$= \frac{\sin\gamma\,(\sec^2\gamma-1)}{\cos\gamma}$$

$$= \tan\gamma\,(\sec^2\gamma-1)$$

$$= (\tan\gamma)(\tan^2\gamma)$$

$$= \tan^3\gamma =$$

✓

6g

$$\frac{\cot^2\theta - 1}{1-\tan^2\theta} \stackrel{?}{=} \cot^2\theta$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cancel{(\cos^2\theta - \sin^2\theta)}}{\sin^2\theta} \cdot \frac{\cos^2\theta}{\cancel{(\cos^2\theta - \sin^2\theta)}}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} =$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

✓

6i

$$\frac{\cos^2 x + 3\cos x + 2}{\sin^2 x} \stackrel{?}{=} \frac{2+\cos x}{1-\cos x}$$

$$= \frac{\cos^2 x + 3\cos x + 2}{1-\cos^2 x}$$

$$= \frac{\cos^2 x + 3\cos x + 2}{(1+\cos x)(1-\cos x)}$$

$$= \frac{(\cos x + 2)\cancel{(\cos x + 1)}}{\cancel{(1+\cos x)}(1-\cos x)}$$

$$= \frac{\cos x + 2}{1-\cos x} =$$

✓

6h

$$\frac{\tan\alpha}{\sec\alpha + 1} \stackrel{?}{=} \frac{1}{\cot\alpha + \csc\alpha}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}}{\left(\frac{1}{\cos\alpha} + 1\right)} \cdot \frac{\cos\alpha}{\cos\alpha}$$

$$= \frac{\sin\alpha}{1+\cos\alpha}$$

$$= \frac{1}{\frac{\cos\alpha}{\sin\alpha} + \frac{1}{\sin\alpha}}$$

$$= \frac{1}{\frac{\cos\alpha + 1}{\sin\alpha}}$$

$$= \frac{\sin\alpha}{\cos\alpha + 1}$$

$$=$$

✓

6j

$$\frac{1 + \sin x + \cos x}{1 + \cos x - \sin x} \stackrel{?}{=} \sec x + \tan x$$

$$= \frac{1+\cos x + \sin x}{1+\cos x - \sin x}$$

$$= \frac{(1+\cos x + \sin x)}{(1+\cos x - \sin x)} \cdot \frac{(1+\sin x)}{(1+\sin x)}$$

$$= \frac{(1+\cos x + \sin x)(1+\sin x)}{1 + \cos x \cancel{- \sin x} \cancel{+ \sin x} + (\sin x)(\cos x) - \sin^2 x}$$

$$= \frac{(1+\cos x + \sin x)(1+\sin x)}{1-\sin^2 x + \cos x + (\sin x)(\cos x)}$$

$$= \frac{(1+\cos x + \sin x)(1+\sin x)}{\cos^2 x + \cos x + (\sin x)(\cos x)}$$

$$= \frac{\cancel{(1+\cos x + \sin x)}(1+\sin x)}{\cos x\,\cancel{(\cos x + 1 + \sin x)}}$$

$$= \frac{1+\sin x}{\cos x} =$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1+\sin x}{\cos x}$$

✓